WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) What is meant by expectation value?

(b) Explain what is meant by spin-orbit coupling.

(c) If $L_{\pm}|1,m\rangle = C_{\pm}|1,m\pm1\rangle$, find $C_{\pm}$. Here $L_{\pm} = L_x \pm iL_y$.

(d) Find the value of the commutator $[\sin(x), p_x]$, where symbols have their usual meanings.

(e) The eigenvalue equations corresponding to two operators A and B are respectively given by $Af(x) = af(x)$ and $Bf(x) = bf(x)$, where a and b are the corresponding eigenvalues of the operators A and B. Prove that the operators A and B commute.

(f) The one-dimensional wave function is given by $\psi(x) = \sqrt{a}e^{-ax}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$.

(g) Consider a particle whose Hamiltonian matrix is $H = \begin{pmatrix} 2 & i & 0 \\ -i & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Is $|\lambda\rangle = \begin{pmatrix} i \\ 7i \\ -2 \end{pmatrix}$ an eigenstate of H ?

(h) What is Larmor precession of electron in an atom?

(i) Two operators A and B have simultaneous eigen-functions. Show that $[A \cdot B] = 0$.

(j) Can the principal quantum number for an electron in a hydrogen atom be zero? Explain your answer.

(k) Justify the statement that the probability current density cannot be directly measured.

(l) What is Lande g-factor? Obtain an expression for it in terms of 1, s and j.

(m) Show that if a quantum particle has the wave function $\psi = e^{ikz}$, the z-component of its angular momentum is zero.

(n) Can Lithium $(Z = 3)$ give rise to normal Zeeman effect? Justify your answer.

2. (a) What is stationary state? If ψ_1 and ψ_2 are two eigen states with energy E_1 and E_2 respectively, check whether the state $(\psi_1 + \psi_2)$ is stationary or not. 1+2

   (b) Does a stationary state evolve with time? Explain your answer. 2

   (c) If $\psi_l^m(\vec{r}, t)$ be the simultaneous eigenfunctions of the angular momentum operator L and L_z, what are the eigenvalue equations corresponding to the operators L^2 and L_z? 3+2

3. (a) The initial state of a two level system is a superposition of the ground state, $|E_1\rangle$, and first excited state, $|E_2\rangle$ (symbols bearing usual meaning), as follows: $|\Psi\rangle = 3|E_1\rangle + 2|E_2\rangle$. 2+2

   (i) Find the possible results of energy measurement with their corresponding probabilities.

   (ii) Find the average value of energy. Will it be time dependent?

   (b) Explain the origin of spin-orbit interaction. 3

   (c) What is meant by space quantization? What role does magnetic quantum number play in space quantization? Explain in the light of vector atom model. 3

4. (a) A particle of mass *m* and momentum *p* is incident from left on the potential step of height V_0. Calculate the probability that the particle is scattered backward by the potential if $\frac{p^2}{2m} < V_0$. 4

   (b) Calculate the expectation value of the potential energy of the electron in the 1s state of H-atom. The wave function of the 1s-electron of H-atom is given by $\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ where a_0 is the Bohr radius $= 4\pi\varepsilon_0\hbar^2/me^2$, with usual meanings of symbols. 3

   (c) Determine stating reasons whether each of the following functions is acceptable or not as a state function over the indicated intervals. 3

   (i) $\sqrt{\frac{2}{l}} \sin\frac{n\pi x}{l}$ in the range -0 to $+l$

   (ii) $\sin^{-1} x$ in the range +1 to – 1.

5. (a) Find the energy of n^{th} state of a linear harmonic oscillator with mass *m* and frequency ω. Show that the average potential energy of the n^{th} state of a linear harmonic oscillator is half of the energy of the oscillator in this state. 3+3

   (b) Discuss the goal of Stern-Gerlach experiment. Why is it necessary to apply an inhomogeneous magnetic field in this experiment? 3+1

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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