WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

STSACOR06T-STATISTICS (CC6)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 5×4 = 20

1. Derive the sampling distribution of $\frac{s_1^2}{s_2^2}$, where s_1^2 and s_2^2 be the sample variances based on independent samples from $\mathcal{N}(\mu_1, \sigma^2)$ and $\mathcal{N}(\mu_2, \sigma^2)$ populations respectively. 5

2. Let $X_1, X_2, \ldots.., X_n$ be a random sample drawn from $R(0, \theta)$, $\theta > 0$. Find the joint pdf of $U = X_{(n)} - X_{(1)}$ and $V = \frac{1}{2}(X_{(1)} + X_{(n)})$. Hence, find the marginal distributions of U and V. Where $R(0, \theta)$ is the rectangular distribution over $(0, \theta)$. 3+2

3. The joint pdf of $(U, V)'$ is given by 3+2

$$f(u, v) = \begin{cases} \exp\left(-\theta u - \frac{v}{\theta}\right) & \text{if } 0 < u, v < \infty \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$. Define $X = UV$ and $Y = U/V$. Find the joint pdf of $(X, Y)'$. Hence find the marginal distributions of X and Y.

4. Let $X, Y, Z \sim iid\ Gamma(\alpha, \beta)$, $\alpha, \beta > 0$. Show that $U = \frac{Y}{X+Y}$ and $V = \frac{Z}{X+Y+Z}$ are independently distributed. Hence find the marginal distributions of U and V. 4+1

5. Let $\{(X_i, Y_i)'; i = 1, 2, \ldots., n\}$ be a random sample from $\mathcal{BN}(0, 0, \sigma^2, \sigma^2, \rho)$ where $\sigma > 0$ and $|\rho| < 1$. Derive, in detail, the sampling distribution of $\overline{U} = \frac{1}{n}\sum_{i=1}^{n} U_i$ where $U_i = X_i/Y_i$, $i = 1, 2, \ldots.., n$. Modify your answer when $\rho = 0$. 4+1

6. What is confidence coefficient in the context of interval estimation? Let $Y_i = \alpha + \beta x_i + e_i$, $i = 1, 2, \ldots\ldots, n$; where x is a non-stochastic variable and $e_i \sim iid\, \mathcal{N}(0, \sigma^2)$, $i = 1, 2, \ldots\ldots, n$ with σ^2 is unknown. Find $100(1-\alpha)\%$ confidence interval of the parameter β. 1+4

GROUP-B

Answer any *two* questions from the following 10×2 = 20

7. (a) On the basis of a random sample of size n drawn from $R(-\theta, \theta)$, where $\theta > 0$, suggest two distinct unbiased estimators of θ. 3

(b) Suppose that a pair of random variables $(X_1, X_2)'$ has the joint pdf given by 7

$$f(x_1, x_2) = \alpha\, \varphi(x_1, x_2\ ;\ \mu, \mu, \sigma^2, \sigma^2, \rho) + (1-\alpha)\, \varphi(x_1, x_2\ ;\ \mu, \mu, \sigma^2, \sigma^2, -\rho)$$

with $0 < \alpha < 1$, where $\varphi(x_1, x_2\ ;\ \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ stands for the bivariate normal pdf with means μ_1, μ_2, variances σ_1^2, σ_2^2 and correlation coefficient $\rho \in (0, 1)$. Show that marginally X_1 and X_2 are normal but $X_1 + X_2$ is not normally distributed.

8. Define Student's t statistic. Derive its sampling distribution. Discuss its use in statistical inference. 1+7+2

9. (a) Let the critical region corresponding to the testing problem $H_0 : X \sim \mathcal{N}(0, 1)$ *vs.* $H_1 : X \sim Cauchy(0, 1)$ is given by 4

$$\omega = \{x \in \mathbb{R} \mid x > 2 \text{ or } x < -2\}.$$

Find probability of type-I error and type-II error. [Given that $\Phi(2) = 0.97725$]

(b) Suppose a paper producing company produces a very good quality paper. The producer claims that the chance of occurring defect(s) in a page is less than 5%. To verify the claim a survey was conducted to collect information regarding the number of defects observed per page. On the basis of a random sample of size n collected from the survey, formulate the testing problem and describe a test procedure to judge the producer's claim. 6

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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