WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Supplementary Examination, 2021

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Find the inverse of the matrix $A=\begin{bmatrix} 2 & 5 \\ 6 & 1 \end{bmatrix}$.

(b) If $A=\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$, find the value $(2A+3B)$.

(c) Show that the vectors (1, 2, 3) and (4, –2, 7) are linearly independent in V_3 over the field F of real numbers.

(d) If $A=\begin{bmatrix} 2 & 1 & 6 \\ 0 & -1 & 2 \\ 5 & 4 & 0 \end{bmatrix}$ then find trace A.

(e) Find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

(f) Prove that all the diagonal terms of a skew-symmetric matrix are zero.

(g) Find the eigen value of $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

(h) State Cayley Hamilton Theorem.

2. (a) If $A=\begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify $(AB)'=B'A'$. 3

(b) Solve if possible, 5

$$x+y+z=1$$
$$2x+y+2z=2$$
$$3x+2y+3z=5$$

by Matrix Inversion Method.

3. (a) Show that $S=\{(x,\, y,\, z)\in R^3 : 3x-4y+z=0\}$ is a sub-space in R^3. 4

(b) Show that the set of vectors {(1, 2, 3), (2, 3, 0), (3, 0, 1)} is a basis of R^3. 4

4. Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. 8

5. (a) Express (–1, 2, 4) as a linear combination of α = (–1, 2, 0), β = (0, –1, 1) and γ = (3, –4, 2) in the vector space V_3 of real numbers. 3

(b) Let the vectors (0, 1, a), (1, a, 1), (a, 1, 0) of the vector space $R^3(R)$ be linearly dependent, then find the value of a. 5

6. (a) Let $T: V \to W$ be a linear transformation such that $N(T)=\{\theta\}$, θ is the null vector of V. If $\alpha_1, \alpha_2, \ldots\ldots, \alpha_r$ form a basis of V then $T(\alpha_1), T(\alpha_2), \ldots., T(\alpha_r)$ also form a basis of $T(V)$. 4

(b) Prove that any orthogonal set of non-null vectors in an inner product space is linearly independent. 4

7. (a) If $A=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, then use Cayley-Hamilton theorem to express $A^6-4A^5+8A^4-12A^3+14A^2$ as a linear polynomial in A. 6

(b) When a matrix is invertible? 2

8. (a) If $A=\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix X, such that $2A+3X=5B$. 4

(b) Find the linear transformation $T:\mathbb{R}^3 \longrightarrow \mathbb{R}^2$ which transforms the vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) to the vectors (1, 1), (2, 3) and (–1, 2) respectively. Hence find T(1, 1, 1). 4

9. (a) Reduce the following quadratic form to normal form and then examine whether the quadratic form is positive definite or not $6x^2+y^2+18z^2-4yz-12zx$. 5

(b) If A be a square matrix , then show that the product of the characteristic roots of A is det A. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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